Written Exam at the
Department of Economics, summer 2020
Economic Growth
Re-exam, August 13, 9am-noon

3-hour open book exam. Answers only in English.

This exam question consists of 5 pages in total.

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- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

1 Short essay questions

Question 1.a

Based on the insights from Hsieh and Moretti (2019) and the other papers on misallocation on the syllabus, discuss the following proposition: Moving public sector jobs from Copenhagen to provincial towns is a good idea. Be clear about the assumptions you make in your answer.

Answer:

If public sector productivity levels follow general productivity levels, and there is perfect migration, it would not affect aggregate productivity or misallocation because the movement of public jobs would be exactly offset by a move in the other direction of private jobs until the economy once more is in equilibrium. Misallocation due to housing constraints would be unaffected. The relative productivity levels of the public and private sector will change, which potentially could affect misallocation of talent between the public and the private sector.
If public sector productivity is independent of geographical location, one could argue that moving public jobs outside Copenhagen is a good idea if the move allows private firms to get access to the higher productivity enviroment in Copenhagen.
This is an open question, so other perspectives from the course can also be discussed by students.

Question 1.b

A key implication of so-called "AK models" of endogenous growth is that a higher investment rate leads to faster growth in the long-run (i.e., in steady state). How well does this prediction fit the data? Suppose a researcher runs the following panel regression for a large sample of countries (indexed by i):

$$\ln y_{it} = \theta_i + u_t + \beta_1 \ln y_{it-1} + \beta_2 \ln s_{it} + \beta_3 x_{it} + \epsilon_{it},$$

where y is GDP per capita, $\theta_i$is a fixed effect, u_t is a time fixed effect, s is the investment rate, x is a vector of control variables and ϵ is a noise term. The index t refers to calendar time. Imagine furthermore that when estimating the model you find $\beta_2 > 0$. Would you see this finding as evidence in favor of the AK-model? Explain why or why not.

Answer:

It might, but you actually cannot say for sure, given the specification. To see this, assume growth in the long-run is exogenous, in keeping with a standard neoclassical growth model. That is, in the long-run the rate of technological change is given by g, and exogenous. In terms of the specification above, changes in u would pick up exogenous technological change, $\Delta u = g$. Consider a steady

state, where the rate of growth is constant. First difference the equation stated in the question, and imposing the steady state requirement (marked by "*´´), gives

$$0 = \Delta y^* = \Delta u + \beta_1 \Delta y^* \Longleftrightarrow \Delta y^* = \frac{g}{1-\beta_1},$$

which is independent of investment (and thus whether β_2 is significant or not). Hence, β_2>0 does not unambigiously lend support to the AK-prediction that a higher investment rate translates into faster growth. Naturally, you CAN write down a theoretical model of endogenous growth where β_2>0 is consistent with its dynamics. A so-called asymptotic AK model would predict that growth is "level dependent", until economic activity becomes "large" and thus be consistent with the stated finding. But you can't know for sure, and hence the stated result does not allow the researcher to distinguish between a situation where s has "levels-effects" (as in neoclassical growth models) or growth effects (as in Ak-type models).

Question 1.c

Pikkety's "second law of capitalism" asserts that the ratio of capital to net-output, κ, in the long run is given by

$$\kappa \equiv \frac{k}{\widetilde{y}} = \frac{\widetilde{s}}{g},$$

where $\widetilde{s}$ is the net savings/investment rate and g is the growth rate of GDP. Net output is defined as $\widetilde{y} = y - dk$. Clearly, if g tends to zero, κ tends to infinity. Is this prediction credible? Why/Why not?

Answer:

There are reasons to be sceptical. Piketty's theory requires that the net savings rate is constant and independent of g. Specifically

$$\dot{k} = \widetilde{s}(y - dk) \equiv s\widetilde{y}$$

where d is the rate of capital depreciation, and s is the net-savings rate (and y-dk is net output, $\widetilde{y}$). In this sort of model, the gross savings rate is endogenous. To derive it, note that

$$g = \widetilde{s}\frac{y}{k} - \widetilde{s}d \Longleftrightarrow \left(\frac{k}{y}\right)^* = \frac{\widetilde{s}}{g + \widetilde{s}d}$$

Next, by definition

$$s = \frac{\widetilde{s}\,(y - dk) + dk}{y} = \widetilde{s} - (1 - \widetilde{s})\, d\frac{k}{y} = \widetilde{s} - \frac{(1-\widetilde{s})\, d\widetilde{s}}{g + \widetilde{s}d}.$$

Hence, in a steady state where g=0 we have that the gross savings rate s = 1! The intuition is that a steady state (actually, a stationary state) implies a constant capital stock, which in turn requires y = dk. That is total capital depreciation exhaust output leaving nothing for consumption. While this is not evidene AGAINST the prediction that κ tends to infinity it does require a very aggressive savings strategy from the consumers, which may sound incredible. It is clear that in a world where people value consumption (such as in a model where individuals derive utility from consumption) this prediction is hard to reconsile with optimizing behavior. For instance, a model with Ramey-consumers will not decide to put consumption to zero. If the student also discuss evidence on the link between savings and growth, this is nice (evidence suggest g and s are positively correlated, and similarly for the net-savings rate and growth; hence, no model uniformly outperforms others).

2 Inflation and new products

In this question you are asked to analyze how the introduction of USB drives affected the measured rate of price inflation in digital storage. Table 1 presents a stylized example with four periods. In period 1, floppy disks were the only form of digital storage. In period 2, USB drives became available, but statistical agencies did not collect data on the number of drives sold until period 3. In period 4, USB drives had completely replaced floppy disks. Prices in the table are per gigabyte (GB), and quantities are likewise measured in gigabytes.

Table 1: Digital storage

Period	1	2	3	4
USB drive price/GB	n/a	4.0	2.5	2.0
USB drives sold (GB)	0	?	20	24
Floppy disk price/GB	3.00	3.15	3.30	n/a
Floppy disks sold (GB)	60	45	20	0

Question 2.a

Derive a price index for digital storage using the linking method. Set the index in period 1 to 100 and use standard chain weights in the calculation (i.e., use expenditure shares in period t-1 to compute inflation between period t-1 and period t). Report the index values for all four years. Explain why this measure of inflation in digital storage prices may be misleading.

Answer:

Period	1	2	3	4
Index	100	105	110	88

This measure may be misleading because it disregards the decline in USB drive prices before period 3. It also disregards any consumer surplus from the new good.

Question 2.b

Derive an approximate Coli again using chain weights (hint: the first step is to use observed prices and quantities to derive the demand curve for USB drives). What problems does this new index solve compared to the linking method? Are there still problems left unsolved?

Answer:

The first step is to derive a price a linear approximagtion of the inverse demand curve. Using the data points in period 3 and 4, we get:

$$P = 5 - 0.125Q$$

From this, the quantity in period 2 can be found as as $Q = 8$, which allows us to calculate expenditure shares for each period. We can similarly estimate the price for floppy disks in period 4 to be $P = 3.42$ by using prices and expenditures for period 2 and 3 (implicitly assuming that demand curves for the two products are unrelated, which is obviously a simplification). By combining expenditure shares with the corresponding price changes in USB drives and floppy disks, the approximate COLI index can be calculated. The results are shown in the table below.

Period	1	2	3	4
Expenditure share USB	0	0.184	0.431	1
Expenditure share Floppy	1	0.816	0.569	0
COLI	100	105.0	101.8	95.2

By calculating an approximate COLI this way, we can now take the decline in prices between period 2 and 3 into account. We still do not capture any gain in consumer surplus from the introduction between period 1 and 2, so the index value in period 2 corresponds to the linked index above. By assuming that the demand curve is linear, however, we may underestimate the quantity sold in period 2, and consequently overestimate inflation. A more worrying problem is that because we are using expenditure weights from period 3 to calculate inflation, we are implicitly giving floppy disks a substantial weight when we calculate inflation between perod 3 and 4.

Question 2.c

Derive an approximate COLI index using expenditure shares from period 3 as weights in all four periods. Comment on the results. What problem does this new index solve compared to the COLI

index with chain weights? Should we consider this new index an improvement over the chain-weighted version, or does it have its own problems?

Answer:

The price in period 1 can be estimated as $P = 5$, and the COLI in this case becomes:

Period	1	2	3	4
Index	100	94.2	81.5	76.2

The rate of deflation is now faster than in the previous question, and the arrival of USB drives now affect measured inflation already in period 2. The problem with this measure is that weights are measured in period 3, so USB drives has a too high weight in the price index for period 2 and 3, meaning that the rate of deflation is overstated. The problem with floppy disks having a large weight when calculating inflation between period 3 and 4 is still not solved.

Question 2.d

Now calculate the rate of inflation (or deflation) between period 1 and period 4 according to the three price indices that you have derived in the previous three questions. Compare the results to the true rate of inflation in the price per unit of storage. Are the results as you would have expected? Can you think of ways to improve the accuracy of estimated inflation beyond what you have done in the three questions above?

Answer

	Inflation
Linked	-12%
Chain-weighted COLI	-5%
Fixed-weight COLI	-24%
True inflation	-33%

Some of the results are perhaps surprising: the linked index actually performs better that the chain-weighted COLI. The reason is, as stated above, that the COLI puts to high weight on floppy discs between period 3 and 4, whereas the linked index entirely uses the rate of deflation for USB drives. Had we observed prices and quantities at a higher frequency, we would be able to observe a more gradual decline in the expenditure share on floppy disks, which would alleviate this problem. In general, higher-frequency re-weighting of inflation indices tends to decrease the upward bias in the measured rate of inflation.

This is an open question, so additional perspectives from the syllabus could also be included in the discussion. Students could, for instance, mention that USB drives are more convenient that floppy disks, meaning that what is labeled as the true rate of inflation here in fact understates the quality-adjusted rate of deflation.

3 Stagnation and the natural rate of interest

Consider an overlapping generations model where economic activity extends into the infinite future. Specifically, we are considering a Diamond-model, where both population growth and exogenous technological change is ignored, and where preferences are logaritmic. Hence

$$u_t = lnc_{1t} + \frac{1}{1+\rho} lnc_{2t+1},$$

and the individual budget constraints are $c_{1t} + s_t = w_t$ and $c_{2t+1} = (1 + r_{t+1})\, s_t$. The notation is standard.

Question 3.a

Show that savings of the young is given by

$$s_t = \frac{1}{2+\rho} w_t.$$

Answer

Substituting the budget constaints into the utility function, differentiating with respect to s and rearranging leads to the stated result.

Question 3.b

The representative firm operates the production technology, $y = f(k)$, where k is capital-labor ratio and y is the output-labor ratio. All markets are competitive, and firm profits can be written $f(k) - (r + d)\, k$. The parameter d is the rate of capital depreciation. The production function, $f\,(k) \equiv F(K/L, 1)$ where F exhibits constant returns to K and L. From now on, and without loss, we normalize the size of the labor force to one, $L = 1$.

Under these assumptions, show that profit maximizing behavior and competitive markets imply

$$r_t = f'\,(k_t) - d$$

and

$$w_t = f\,(k_t) - f'\,(k_t)\, k_t.$$

Answer

Differentiating profits wrt k gives the first FOC. The second follows since CRTS implies absence of pure profits. This requires w=f(k))-(r+d)k. Inserting the FOC provides the second condition.

Question 3.c

Since individuals consume their total savings during old age the future capital stock only depends on the savings of the currently young, $k_{t+1} = s_t$. It has been used that the size of the labor force (and thus the size of the young generation) is normalized to one.
From now on we assume f'>0, f"<0. A standard requirement for endogenous growth to arise is that the marginal product of capital is bounded away from zero

$$lim_{k\to\infty} f'(k) = A > 0.$$

Show that this technology assumption leads to (asymptotic) stagnation in the present model. Why does positive exponential growth not arise, even though we have imposed the standard requirement for endogenous growth?

Answer

Using the information given in the question we have the following

$$k_{t+1} = \frac{1}{2+\rho} w_t = \frac{1}{2+\rho} \left(f\left(k_t\right) - f'\left(k_t\right) k_t \right)$$

This means the growth rate can be written

$$\frac{k_{t+1}}{k_t} = \frac{1}{2+\rho} \left(\frac{f\left(k_t\right)}{k_t} - f'\left(k_t\right) \right)$$

Hence

$$lim_{k\to\infty} \frac{k_{t+1}}{k_t} = \frac{1}{2+\rho} \left(lim_{k\to\infty} \frac{f\left(k_t\right)}{k_t} - lim_{k\to\infty} f'\left(k_t\right) \right) = 0$$

as $lim_{k\to\infty} \frac{f(k)}{k} = f'\left(k\right) = A$ using L'Hopital's rule. Consequently, the capital stock will ultimately stagnate. (recall that the condition for a steady state is that $k_{t+1} = k_t \Rightarrow k_{t+1}/k_t = 1$ and that the law of motion is fulfilled. The above argument tells us that there exist a finite k such that a stationary state emerges where $k_{t+1}/k_t = 1$.)
The requirement for endogenous growth only speaks to feasibility, not to whether (positive) growth arises in equilibrium. In the present case, preferences are such that individuals save a constant fraction of wage income. The requirement for endogenous growth (which curbs diminishing returns

to capital in the limit) means that the share of wage income tends to zero. Consequently, savings tend to zero (in proportion to the economy!) and capital accumulation therefore eventually ceases.

Question 3.d

The "natural real rate of interest" is usually defined as the real rate which is consistent with stable inflation and output at its structural level. Accordingly, the steady state predictions of the present model carries predictions regarding the long run movements in the natural rate of interest.

(a) How has the real rate of return changed over the last three decades empirically? (b) What could, within the present model, account for such changes?

Answer

As we have seen, the model admits a stationary state where $k_{t+1} = k_t = k^*$. The real rate of interest in the steady state is given by

$$r^* = f'(k^*) - d.$$

Hence, factors that serve to increase the steady state capital stock (e.g., a higher propensity to save, $\rho \uparrow$)will work to reduce the natural rate (since we have diminishing return being operative). One explanation for the low real rates are in fact higher savings. (a variety of causes have been debated – here it is only caputed crudely by a preference parameter). Another reason is higher depreciation, d: if the rate of depreciation has increased, the net marginal product of capital declines, which should work to reduce the natural rate.

Question 3.e

An interesting empirical finding is that the natural real rate recently seems to have ventured into negative territory. Is a negative natural rate a possibility in the present model? Explain.

Answer

Yes, it is feasible. Since d does not influence capital accumulation it follows from the FOC that $r^*<0$ is feasible. Natually, for the economy to "work" we would need to ensure $1+r^*$ is non-negative, otherwise it will mean the old starve to death. But as long as $1+r^*>0$ the only effect of higher depreciation (beyond negative real rates) is that the consumption of the old is reduced. Hence, an OLG model can account for the seemingly puzzling phenomena of negative real rates.